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# Building the standard model on a D3-brane

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Abstract: We motivate and apply a bottom-up approach to string phenomenology, which aims to construct the Standard Model as a decoupled world-volume theory on a D3-brane. As a concrete proposal for such a construction, we consider a single probe D3-brane on a partial resolution of a del Pezzo 8 singularity. The resulting world-volume theory reproduces the field content and interactions of the MSSM, however with a somewhat extended Higgs sector. An attractive feature of our approach is that the gauge and Yukawa couplings are dual to non-dynamical closed string modes, and are therefore tunable parameters.

Keywords: Intersecting branes models, Superstring Vacua.

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## Contents



## 1. Introduction: bottom-up string phenomenology

String theory has presented itself with the formidable task of taming the quantum realm of gravity, while simultaneously furnishing a predictive and testable theory of particle physics. In attempting to meet this dual challenge, string phenomenology traditionally adopts a "top-down" point of view, which aims to construct realistic compactification scenarios starting from the full 10-dimensional closed string theory [1, 2], possible augmented with one or more D-branes [3]. The thrust of this approach is that, by simultaneously controlling and scanning both the string scale geometry and the low energy field theory, one can isolate realistic backgrounds that meet all consistency requirements at both ends. Recent progress in mapping out the "closed string theory landscape," the vast collection of potentially stabilized string vacua [4], has strengthened the belief that such consistent backgrounds indeed exist, possibly even in abundance [5]. Finding a single one of them, however, still seems far too challenging a task at present.

In a complementary development, fueled by the deepened understanding of string dualities and D-brane physics, open string theory has evolved into a remarkably successful tool for building 4-d supersymmetric field theories. In particular, it is now realized that by taking a judicious low energy limit of the world-volume theory on $N$ D3-branes, one recovers a purely 3+1-dimensional gauge theory, decoupled from gravity and higher dimensional dynamics [6]. In this decoupling limit, the closed string background gets frozen into a set of non-dynamical, and thus largely tunable, gauge invariant couplings. By placing one or more D-branes near various types of geometric singularities, realizations of large classes of gauge theories have been uncovered [7 − 14]. Evidently, open string theory has become the preferred duality frame for representing weakly coupled, as well as strongly coupled, 4-d quantum field theories.

Given this rich "open string theory landscape," it is a well-motivated question whether, with currently available technology, one can find an explicit realization of the supersymmetric Standard Model as the world-volume theory on one or more D3-branes. Since every decoupled theory, via its space of tunable couplings, stretches out over a sizable open neighborhood within the space of 4-d field theories, one can even aim to reproduce the spectrum and couplings within phenomenological bounds. Though clearly a non-trivial challenge, this question is still far less ambitious, and thus easier to answer, than finding a fully realistic closed string compactification. But it would be a useful first step: only after one knows how to represent the observed particle spectrum as an open string theory near a suitable singularity, one can start to look for compact geometries that contain this singularity. We thus view the bottom-up approach to string phenomenology [15 − 23] as a promising route towards unlocking some of the mysteries of the closed string theory landscape.

The experimental fact that guides the bottom-up perspective is the exponential separation between the TeV scale of particle physics and the Planck scale of quantum gravity. Warped compactifications assumption that all low energy physics takes place in a highly red-shifted region of the internal geometry. This geometric viewpoint thus naturally places the Standard Model on a world-brane near the apex of a warped throat.

A key manifestation of the gauge hierarchy is that, at the TeV scale, one can make a clean separation between dynamical gauge and matter fields and non-dynamical coupling constants — even though both start out as equally dynamical degrees of freedom in the full high energy string theory. It is thus accurate, and even appropriate, to isolate the low energy worldbrane physics from the closed string dynamics, by taking a decoupling limit in which the Planck scale is sent off to infinity. In geometric terms, this limit replaces the finite warped throat region by an infinite, non-compact Calabi-Yau singularity.

Due to the interaction with the ambient geometry, a D3-brane on a CY singularity breaks up into various fractional branes. As a result, its world-volume theory takes the non-trivial form of a quiver gauge theory: it has one $\mathrm{U}(n_i)$ gauge multiplet for each constituent fractional brane and bifundamental chiral matter associated with each brane intersection [7, 11, 25 − 38]. In the decoupling limit, the gauge invariant coupling constants of this quiver gauge theory are determined by non-dynamical asymptotic boundary conditions on the closed string fields, and can thus be viewed as continuously tunable parameters.

This paper is our first progress report on our search for a D3-brane realization of



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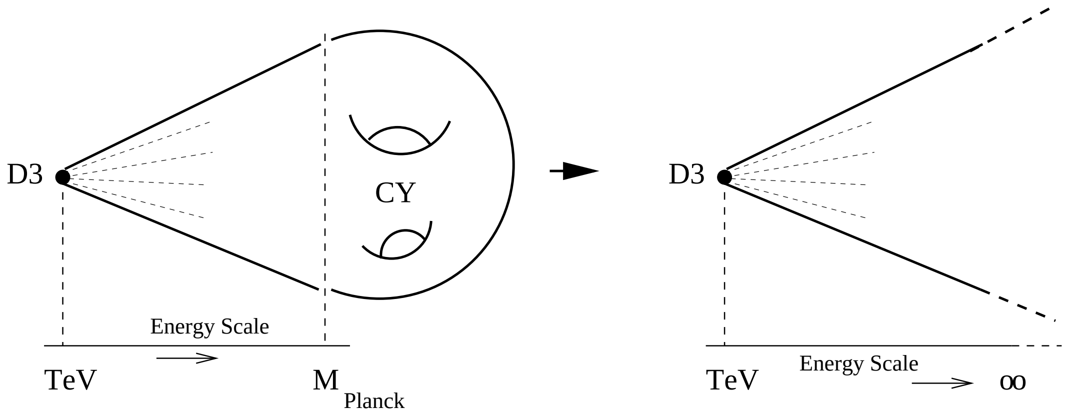


**Figure 1:** Our bottom-up approach to string phenomenology assumes that the Standard Model is localized on a D3-brane at the apex of a highly warped throat. The D3-brane theory is accurately described via a decoupling limit, in which the Planck scale is sent off to infinity, leaving behind a non-compact Calabi-Yau singularity.



the Standard Model. Our strategy is as follows. We start by setting up the rules that define quiver gauge theories, and introduce a corresponding minimal quiver extension of the MSSM, which we call the MQSSM. Our target is to find its geometric dual. Since it is hard to immediately guess the right geometry, we first identify a class of Calabi-Yau singularities, such that the probe D3-brane theory is just large enough to contain the MSSM, and has a rich enough space of couplings and vacua to allow the necessary tuning. We then look for a suitable symmetry breaking process towards the MQSSM quiver theory. After translating into the dual geometric language, the symmetry breaking amounts to into a specific partial resolution of the CY singularity, which then provides the sought after geometric dual. To go further one must turn on various soft supersymmetry breaking terms. Apart from some general comments, we leave this problem for the future.

The specific class of geometries we will consider are the del Pezzo 8 singularities. By following the outlined procedure, we identify a specific partial resolution of the del Pezzo 8 geometry for which the D3-brane gauge theory has the Standard Model gauge group, $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, and matter content, three families of quarks and leptons with all the right charges, plus a somewhat extended Higgs sector. All matter fields appear with the proper chiralities and all have classically tunable Yukawa couplings. The quiver diagram of the model is given in figure 8. In the final section, we discuss some physical aspects of the MQSSM and address some possible criticisms of our approach.

## 2. A quiver extension of the MSSM

It will be useful to introduce a minimal quiver gauge theory extension of the supersymmetric Standard Model. The motivation for presenting it is two-fold: (i) it will help with recognizing, among the vast collection of possibilities, those open string theory constructions that may contain the MSSM as a special limit, and (ii) it will give a useful preview of typical extra features that arise in generic open string set-ups, and that need to be dealt

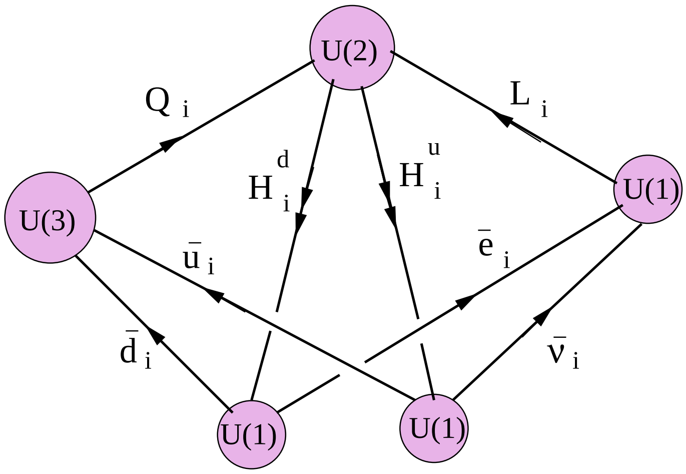


**Figure 2:** The MQSSM is the minimal quiver extension of the Standard Model, as obtained via the rules (a)-(e).

with in making a fully realistic model. For both reasons, let us adopt the quiver diagrammatic rules that apply to D3-branes on CY singularities. These are (see next section):

(a) Each node of the quiver represents a gauge multiplet with $\mathrm{U}(k)$ gauge symmetry.

(b) Each oriented line between two nodes represents a bi-fundamental chiral multiplet.

(c) There is an equal number incoming and outgoing lines connected to every node.

As we will see, these characteristics all have a direct geometric origin. Rule (c) in particular ensures the absence of non-Abelian gauge anomalies for the case of multiple D3-brane probes. Two additional rules, that apply to an especially convenient class of D3-brane configurations, known as "exceptional collections," are:

(d) There are no lines that begin and end at the same node.

(e) There is only one type of oriented lines between any pair of nodes.

Rule (d) excludes the presence of adjoint matter multiplets. Rule (e) states that all bifundamental matter multiplets are purely chiral.

The minimal quiver extension of the MSSM, drawn by using the five rules (a) through (e), is given in figure 2. It depicts all the gauge charges of the fields, while each closed triangle of the diagram represents a possible Yukawa coupling. We see that relative to the MSSM, there are several extra $\mathrm{U}(1)$ factors, and a number of extra Higgs doublets: two pairs for each generation. The additional Higgses are forced on us by rule (c) and the requirement of having all the expected supersymmetric Yukawa couplings

Let us briefly discuss the $\mathrm{U}(1)$ factors. We call the node on the right $\mathrm{U}(1)_0$ and the two nodes at the bottom $\mathrm{U}(1)_u$ and $\mathrm{U}(1)_d$. The five $\mathrm{U}(1)$ generators are denoted by $\{Y_0, Y_1^u, Y_1^d, Y_2, Y_3\}$. The charges of the matter fields are given in the table below.

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We note that the total sum $Q_{\rm tot} = \sum_i Y_i$ automatically decouples: none of the fields is charged under $Q_{\rm tot}$. Of the remaining four generators, some are anomalous. The anomalies cancel for the combinations

$$B - L = \frac{1}{3}Y_3 - Y_0 \qquad\qquad Y = \frac{1}{2}(Y_1^d - Y_1^u - Y_0) + \frac{1}{6}Y_3 \tag{2.1}$$

Obviously, only $\mathrm{U}(1)_Y$ represents an actual local symmetry in the MSSM. The $\mathrm{U}(1)_{B-L}$ symmetry has the desirable consequence that it helps suppress the rate of proton decay, but obviously, it needs to be spontaneously broken at an energy scale of order of a TeV or larger. A natural symmetry breaking mechanism would be to assume a non-zero vacuum expectation value for the bosonic superpartners of the right-handed neutrino fields, which would also mesh well with the small neutrino Yukawa couplings.

| | $Y_0$ | $Y_1^d$ | $Y_1^u$ | $Y_2$ | $Y_3$ | Y |
|---|---|---|---|---|---|---|
| Q | 0 | 0 | 0 | -1 | 1 | $\frac{1}{6}$ |
| $\overline{u}$ | 0 | 0 | 1 | 0 | -1 | $-\frac{2}{3}$ |
| $\overline{d}$ | 0 | 1 | 0 | 0 | -1 | $\frac{1}{3}$ |
| L | 1 | 0 | 0 | -1 | 0 | $-\frac{1}{2}$ |
| $\overline{\nu}$ | - 1 | 0 | 1 | 0 | 0 | 0 |
| $\overline{e}$ | - 1 | 1 | 0 | 0 | 0 | 1 |
| $H^u$ | 0 | 0 | -1 | 1 | 0 | $\frac{1}{2}$ |
| $H^d$ | 0 | -1 | 0 | 1 | 0 | $-\frac{1}{2}$ |

**Table 1:** The U(1) charges

The remaining two U(1)'s indeed have mixed anomalies. In string theory realizations of quiver theories, these are canceled via a generalized Green-Schwarz mechanism. Moreover, via the coupling to the RR-forms, the corresponding gauge bosons typically acquire a mass of order the string scale. We will give a brief outline of this mechanism in the last section. From the low energy perspective, these U(1)'s thus survive as anomalous global symmetries, that, among other things, forbid the presence of $\mu$-terms in the classical superpotential.[1]

In general, the presence of extra U(1) factors as well as extra Higgs fields is characteristic of many string theoretic models. Both are acceptable extensions of the Standard Model, provided the masses and couplings are tuned to satisfy the appropriate phenomenological bounds. For now, however, we postpone the discussion of these issues: instead we set out to find a Calabi-Yau singularity such that the world-volume theory of a probe D3-brane reproduces the MQSSM, the quiver gauge theory of figure 2.

## 3. D3-brane on a del Pezzo 8 singularity

In this section we introduce the quiver gauge theory on the world volume of a D3-brane on a del Pezzo 8 singularity. This quiver theory has been previously derived in [35] using a geometric description, and in this section we begin by recalling some elements of that construction.[2] The geometric description is strictly only accurate for the holomorphic F-term data of the quiver theory. It relies on the fact that the (complexified) Kähler moduli only appear in the D-terms, so that we can extrapolate to large volume without affecting the F-terms. In the large volume limit, topological data of the quiver gauge theory, such

[1] These will have to arise from some other source, such as the Giudice-Masiero mechanism [39].

[2] More details and additional work on D-branes at Calabi-Yau singularities can be found the original literature [27 – 33, 37, 34 – 36, 38].



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as the gauge group, the number and representations of the matter multiplets, as well as the holomorphic superpotential, can be accurately obtained via the geometrical methods outlined below.

### 3.1 Geometry of del Pezzo 8

A del Pezzo surface is a manifold of complex dimension 2, with a positive first Chern class. It is labelled by an integer $n$; the $n$-th del Pezzo surface $\mathcal{B}_n$ can be represented as either $\mathbf{P}^2$ blown up at $n \leq 8$ generic points, or as $\mathbf{P}^1 \times \mathbf{P}^1$ blown up at $n-1$ points. We choose the first representation. Here blowing up a point means replacing it by a sphere. By placing the canonical line bundle over $\mathcal{B}_n$, one obtains a non-compact Calabi-Yau three-fold. In the limit where the Del Pezzo surface shrinks to zero size, one obtains a singular three-fold. We will call this the del Pezzo $n$ singularity.

The second Betti number $b_2(\mathcal{B}_n)$ is equal to $n+1$. A basis of $H_2(\mathcal{B}_n, \mathbf{Z})$ is given by the hyperplane class $H$ in $\mathbf{P}^2$ plus one generator $E_i$ for each of the $n$ blown up points. The generators $E_i$ are called exceptional curves. The intersection numbers are given by

$$H \cdot H \,=\, 1, \qquad E_i \cdot E_j = -\delta_{ij}, \qquad H \cdot E_i = 0\,. \tag{3.1}$$

The canonical class of the del Pezzo surface is

$$K = -3H + \sum_{i=1}^{n} E_i \tag{3.2}$$

It has self intersection $K \cdot K = 9 - n$. The first Chern class of $\mathcal{B}_n$ is $c_1(T\mathcal{B}_n) = -K$. The characteristic property of a del Pezzo surface is that $c_1$ is ample, that is, it has positive intersection with every effective curve on $\mathcal{B}_n$. This in particular implies that $K$ must have positive self-intersection, which gives the restriction $n \leq 8$.

In this paper, we will mostly consider the 8-th del Pezzo surface $\mathcal{B}_8$. It can be constructed as a hypersurface of degree six in the weighted projective space $\mathbf{WP}^3_{1,1,2,3}$ with homogeneous coordinates $(x, y, z, w)$, defined via an equation of the generic form

$$w^2 = Az^3 + By^6 + Cx^6 + \ldots \tag{3.3}$$

This defines a del Pezzo surface because the sum of the weights exceeds the degree of the surface. The 2-d homology of $\mathcal{B}_8$ is generated by 8 exceptional curves $E_i$, corresponding to the 8 blow up points, and the hyperplane class $H$. Via the intersection pairing (3.1), $H_2(\mathcal{B}_8, \mathbf{Z})$ takes the form of an integral lattice in $\mathbf{R}^9$. It has the remarkable property that the 8-dimensional degree zero sub-lattice, defined as those elements with zero intersection with $c_1 = -K$, is even and unimodular. In other words, it is isomorphic to the root lattice of $E_8$. The 8 simple roots, all with self-intersection -2, can be chosen as follows

$$\alpha_i = E_i - E_{i+1}, \qquad i = 1, \ldots, 7 \qquad\qquad \alpha_8 = H - E_1 - E_2 - E_3 \tag{3.4}$$

Combined with $K$, they form a complete basis of $H_2(\mathcal{B}_8, \mathbf{Z})$, with intersection form

$$\alpha_a \cdot \alpha_b = -A_{ab}\,, \qquad\qquad K \cdot \alpha_a = 0 \tag{3.5}$$

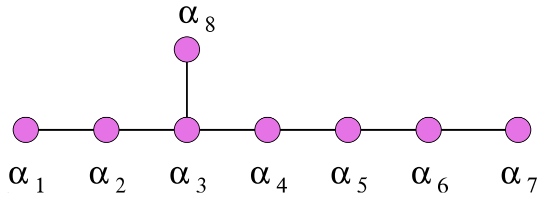


**Figure 3:** The 2-cycles $\alpha_i$ are identified with nodes on the $E_8$ Dynkin diagram.

with $A_{ab}$ the Cartan matrix of $E_8$. The 2-cycles $\alpha_i$ can thus be identified with nodes on the $E_8$ Dynkin diagram, as drawn below. This identification between 2-cycles and simple roots gives rise to a natural action of the Weyl group of $E_8$ on $H_2(\mathcal{B}_8, \mathbf{Z})$, in terms of global diffeomorphisms on $\mathcal{B}_8$ that exchange the exceptional curves while preserving $K$. The Weyl reflections in the simple roots $\alpha_i$ with $i \leq 7$ are simply diffeomorphisms that interchange two of the blown up points, while keeping the rest of the surface fixed. The reflection in $\alpha_8$ looks more complicated, but can be understood in a similar manner.

The ellipses in the homogeneous equation (3.3) represent additional terms that deform the complex structure of the del Pezzo 8 surface. An elegant description of this space of complex structure deformations has been given in [40]. It is built on the observation that the homogeneous eqn (3.3) can chosen of the form

$$w^2 = 4z^3 - g_2\, zy^4 - g_3\, y^6 \;+\; P(x, y, z), \tag{3.6}$$

where $P(x, y, z)$ is a suitable homogeneous polynomial [40] that vanishes at $x = 0$. The degree one surface $x = 0$ is an anti-canonical divisor, and defines an elliptic curve $\mathcal{E}$, given by a Weierstrass equation in $\mathbf{WP}^2_{1,2,3}$. The space of continuous complex structure deformations that keep $\mathcal{E}$ fixed is 8 dimensional.[3] Natural coordinates on this space are the parameters that specify the polynomial $P(x, y, z)$.[4]

Later on, in our construction of a Standard Model-like gauge theory, we will consider a special degenerate limit of the del Pezzo 8 geometry, in which some of the 2-cycles $\alpha_i$, given in eqn (3.4), become effective curves on the del Pezzo surface. The del Pezzo surface then develops a singularity of the appropriate A-D-E type. The maximally degenerate surface of this type is obtained by setting $P(x, y, z) = 0$ in (3.6). The resulting surface is an elliptic singularity of type $E_8$. More generally, one can get an $H$-type singularity for every subgroup $H$ of $E_8$.

[3] A direct way to interpret these 8 complex structure deformations is that they parameterize the locations of 4 of the 8 points that are blown-up to produce $\mathcal{B}_8$ from $\mathbf{P}^2$. The positions of the four other points do not give rise to complex structure moduli, since they can be held fixed by using the $PGL(3, \mathbf{C})$ group of coordinate transformations of the underlying $\mathbf{P}^2$.

[4] As it turns out [40], these describe homogeneous coordinates on the weighted projective space: $\mathbf{WP}^8_{1,2,2,3,3,4,4,5,6}$. Remarkably, the set of weights of the projective space coincides with 1 plus the set of Dynkin labels of the highest coroot of the $E_8$ Kac-Moody algebra. As explained in [40], the above embedding of $\mathcal{E}$ inside the del Pezzo 8 surface induces an $E_8$-bundle over $\mathcal{E}$. The construction can thus be used to establish an isomorphism between the space of complex structure deformations of $\mathcal{B}_8$ and the moduli space of $E_8$ bundles over $\mathcal{E}$.

### 3.2 Quiver gauge theory of a D3-brane on del Pezzo 8

Let $\mathcal{X}$ denote a non-compact Calabi-Yau manifold given by a complex cone over a collapsing del Pezzo 4-cycle $\mathcal{B}$. The D3-brane configurations that we will consider are brane-worlds that fill the 3+1 flat directions, and therefore are localized at a point in $\mathcal{X}$. In the strongly curved background at the tip of the cone, the D3-brane will typically split into several so-called fractional branes that wrap vanishing cycles in $\mathcal{X}$.

As far as the F-terms is concerned, we may blow up the vanishing cycles and perform computations in the large volume limit. From a large volume perspective, the geometric characterization of a fractional brane is as a sheaf $F_i$, which one can think of as a bundle supported on the collapsing del Pezzo surface. The RR-charges of a sheaf $F_i$ are combined in the charge vector

$$\mathrm{ch}(F_i) = (\mathrm{rk}(F_i), c_1(F_i), \mathrm{ch}_2(F_i)), \tag{3.7}$$

which specifies the (D7,D5,D3) charge of $F_i$. The D7 charge is called the rank $\mathrm{rk}(F_i)$ of the sheaf, while the D5 charge is equal to the first Chern class $c_1(F_i)$ and specifies a two-cycle around which the D5-component of the fractional brane is wrapped. If we think of a fractional brane $F_i$ with non-zero rank as a 7-dimensional gauge theory on a D7-brane, $c_1(F_i)$ indicates the presence of non-trivial magnetic flux supported on the corresponding 2-cycles, and $\mathrm{ch}_2(F_i)$ represents a non-trivial instanton number.

In this language, the D3-brane itself is naturally represented as a sky-scraper sheaf $\mathcal{O}_p$ localized at a single point $p$. It splits up in a collection of fractional branes $F_i$, each with integer multiplicities $n_i$, such that the charge vectors of all fractional branes add up to that of a single D3-brane

$$\sum_i n_i \, \mathrm{ch}(F_i) = \mathrm{ch}(\mathcal{O}_p) = (\, 0 \, , \, 0 \, , \, 1 \,) \tag{3.8}$$

To satisfy this condition, some of the charges have to be negative, since all charges associated with 4- and 2-cycles would have to add up to zero. If $n_i$ is negative, it doesn't necessarily mean that it counts anti-branes. In the limit when the del Pezzo surface collapses, the central charge vectors will line up after taking the small volume limit, so that all fractional branes preserve the same four supersymmetry charges.

Each type of fractional brane, with multiplicity $n_i$, contributes a $\mathrm{U}(|n_i|)$ factor to the total gauge group of the world-volume theory. The corresponding $\mathcal{N} = 1$ gauge multiplet is furnished by the lightest modes of open strings with end-points on the same type of fractional brane. The massless spectrum of open strings that stretch between two different types of fractional branes $F_i$ and $F_j$ represent chiral multiplets that transform in the bi-fundamental representation of the corresponding $\mathrm{U}(|n_i|) \times \mathrm{U}(|n_j|)$ gauge group. In case the branes are space filling, i.e. have support on the whole Calabi-Yau, the massless modes correspond to elements of the cohomology of the Dolbault operator acting on the space of bi-fundamental valued anti-holomorphic forms, $\Omega^{(0,\cdot)}(F_i^*, F_j)$. The number of bi-fundamental fields is therefore counted by the proper generalization to sheaves of the cohomology group $H^{(0,\cdot)}(F_i^* \otimes F_j)$, known as the Ext groups $\mathrm{Ext}^k(F_i, F_j)$. Since our fractional branes are not space-filling, we instead need to distinguish between a sheaf $F$, living on the del Pezzo 4-cycle $\mathcal{B}$, and the associated push-forward $i_*F$ on the Calabi-Yau $\mathcal{X}$, which can be thought

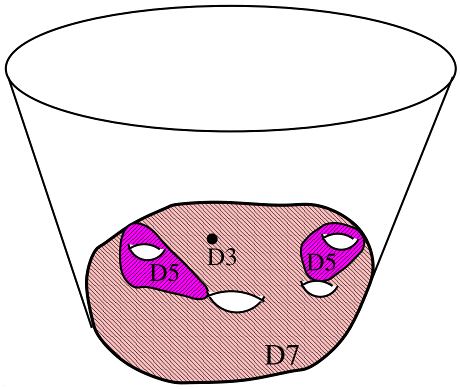


**Figure 4:** D7, D5 and D3-branes wrapped 4-, 2- and 0-cycles of the internal manifold.

of as $F$ extended by zero on $\mathcal{X}$. Taking this into account, one concludes[5] that for each generator of $\mathrm{Ext}^p_{\mathcal{B}}(F_j, F_k)$, one has exactly one chiral field in four dimensions. This is all we need to know for now.

On a given Calabi-Yau singularity, there are different possible choices of basis for the fractional branes. The type of basis that is most well-understood are the so-called "exceptional collections". These satisfy the special criteria that

(i) $\mathrm{Ext}^m(F_i, F_i) = 0$ for $m > 0$. This implies the absence of adjoint matter for a collapsing del Pezzo.

(ii) there exist an ordering of the $F_i$'s, such that $\mathrm{Ext}^m(F_i, F_j) = 0$ for all but one $m$ if $j > i$ and for all $m$ if $i > j$.

The second condition (ii) implies that the bi-fundamental multiplets between any two given nodes has only one type of chirality.

Let us specialize to the case of the del Pezzo 8 singularity. The total homology of $\mathcal{B}_8$ is 11-dimensional; we thus expect to find 11 types of fractional branes. Mathematicians have identified a natural choice of basis of coherent sheaves on del Pezzo singularities, known as three-block exceptional collections. These divide up into three groups, with the special

[5] In short, the argument that leads to this conclusion is as follows. Let us denote the normal bundle of the collapsing cycle by $N$. Then the spectrum of massless modes is counted by [41, 34]

$$\mathrm{Ext}^r_{\mathcal{X}}(i_* F_j, i_* F_k) = \sum_{p+q=r} \mathrm{Ext}^p_{\mathcal{B}}(F_j, F_k \otimes \Lambda^q N). \tag{3.9}$$

For a del Pezzo surface, the normal bundle is equal to the canonical line bundle, $N = K$. Given a generator for $\mathrm{Ext}^p_{\mathcal{B}}(F_j, F_k)$, we can use Serre duality on $\mathcal{B}$ to get a generator in $\mathrm{Ext}^{2-p}_{\mathcal{B}}(F_k, F_j \otimes K)$, hence we get *two* Ext generators on the Calabi-Yau $\mathcal{X}$. These two generators are in turn related by Serre duality on $\mathcal{X}$, which maps $\mathrm{Ext}^p_{\mathcal{X}}(i_* F_j, i_* F_k)$ isomorphically to $\mathrm{Ext}^{3-p}_{\mathcal{X}}(i_* F_k, i_* F_j)$. There is a simple physical interpretation for this doubling. The degree (mod 2) of the Ext group is related to 4-d chirality through the GSO projection. Two generators related by Serre duality therefore have opposite chirality and opposite bifundamental charge, and so they give rise to a particle and its corresponding antiparticle. Since by convention chiral superfields contain a left-handed spinor, the dual pair of generators gives a single chiral superfield in four dimensions – the second generator descends to the conjugate anti-chiral superfield.

property that the intersection pairing between elements of the same block vanishes. The associated quiver diagram thus always has a triangular structure. Exceptional three block collections for the $\mathcal{B}_8$ singularity have been constructed in [42]. The one that is closest to our needs is denoted as type (8.1) in [42]. Unfortunately the actual collection as given in [42] turns out not to be exceptional — probably due to a trivial calculation error. Instead we will pick collection (8.2) in [42] and apply Seiberg dualities (for a short description, see subsection 3.5) until we end up with a quiver of type (8.1). The resulting charge vectors of this collection are:

$$\begin{array}{llll} \mathrm{ch}(F_i) = (1, H - E_i, 0) & \mathrm{i}=1,\,.\,\,,4 & \mathrm{ch}(F_{11}) = & \left(6, -3K + 2\sum\limits_{i=5}^{8} E_i, \frac{1}{2}\right) \\ \mathrm{ch}(F_i) = (1, -K + E_i,\, 1) & \mathrm{i}=5,\,.\,\,,8 & & \\ \mathrm{ch}(F_9) = (1, 2H - \sum\limits_{i=1}^{4} E_i, 0) & & \mathrm{ch}(F_{10}) = & \left(3, -K + \sum\limits_{i=5}^{8} E_i, -\frac{1}{2}\right) \end{array} \tag{3.10}$$

We see that all fractional branes have non-zero D7 and D5-brane components. The D5 branes are wrapped around the 2-cycles as indicated. From this collection, we wish to obtain the quiver gauge theory associated with a single D3-brane. The condition (3.8) that all charge vectors must add up to (0,0,1) yields the following multiplicities

$$n_i = 1\,, \quad i = 1, .\,, 9\,, \qquad\qquad n_{10} = 3\,, \qquad\qquad n_{11} = -3 \tag{3.11}$$

So the gauge theory on the D3-brane has gauge group $\mathrm{U}(3)^2 \times \mathrm{U}(1)^9$.

To obtain the matter content we must determine the dimension of the relevant Ext groups. Since for each pair of sheaves $F_i$ and $F_j$ of an exceptional collection, only one of the Ext groups is non-zero, one can determine its dimension by computing the corresponding Euler character

$$\chi(F_i, F_j) = \sum_k (-)^k \dim\, \mathrm{Ext}(F_i, F_j) \tag{3.12}$$

which can be computed using the Riemann-Roch formula

$$\chi(F_i, F_j) = \int_{\mathcal{B}} \mathrm{ch}(F_i^*)\, \mathrm{ch}(F_j)\, \mathrm{Td}(\mathcal{B}). \tag{3.13}$$

Here $\mathrm{ch}(F_i) = (\mathrm{rk}{+}c_1{+}\mathrm{ch}_2)(F_i)$ denotes the Chern character of $F_i$ and $\mathrm{Td}(\mathcal{B}) = 1 - \frac{1}{2}K + H^2$ is the Todd class of the base $\mathcal{B}$. For exceptional collections, this formula gives an upper triangular matrix with all 1's on the diagonal. Hence we loose no information by anti-symmetrizing:

$$\chi_-(F_i, F_j) = \chi(F_i, F_j) - \chi(F_j, F_i) = \mathrm{rk}(F_i)\deg(F_j) - \mathrm{rk}(F_j)\deg(F_i)\,. \tag{3.14}$$

where $\deg(F_i)$ is the degree of the sheaf, defined as the intersection between the first Chern class of $F_i$ with the canonical bundle of the del Pezzo surface

$$\deg(F_i) = -c_1(F_i) \cdot K\,. \tag{3.15}$$

Formula (3.14) counts, with orientation, the number of intersections within $\mathcal{X}$ between the 2-cycle components of one sheaf with the 4-cycle component of the other. Geometrically, one expects the massless open string states to appear whenever two branes intersect at a point. In terms of the quiver gauge theory, the matrix $\chi_-$ indeed represents the adjacency matrix that counts the number of lines between the nodes. Moreover we see that it can be computed very simply from the charge vectors of the fractional branes.

We have outlined the procedure for obtaining the quiver data for a given exceptional collection of fractional branes $F_i$. The ensuing rules for drawing the quiver diagram are (a) through (e) given in the previous section. Rules (a) and (b) are clear, and rules (d) and (e) represent the special conditions that define an exceptional collection. Rule (c) is a consequence of the geometric fact that each fractional brane consists of 0, 2, or 4-cycles only, and therefore has, on the 6-manifold $\mathcal{X}$, zero intersection with a 0-cycle, i.e. with some isolated point $p$. In other words, the intersection pairing between $F_i$ and the sky-scraper sheaf $\mathcal{O}_p$ that represents a D3-brane located at $p$ vanishes. Using (3.8), this implies

$$\sum_j n_j\, \chi_-(F_i, F_j) = 0 \tag{3.16}$$

for all $i$. This is rule (c). It in particular ensures that, for the case of multiple D3-brane probes, each node is free on non-Abelian gauge anomalies.

We can now obtain the full quiver gauge theory for the collection (3.10). Using (3.14), we obtain the intersection numbers

$$\begin{aligned} \chi(F_{11}, F_i) &= 1\,, \qquad\qquad i = 1, ., 9 \\ \chi(F_i, F_{10}) &= 1\,, \\ \chi(F_{10}, F_{11}) &= 3. \end{aligned} \tag{3.17}$$

The resulting quiver diagram is given in figure 5. We recognize the characteristic form of a quiver gauge theory that follows from a three-block exceptional collection.

### 3.3 D3-brane on $\mathbf{C}^3/\Delta_{27}$ orbifold

As it turns out, the above quiver diagram is identical to that of the D3-brane theory on the $\mathbf{C}^3/\Delta_{27}$ orbifold singularity. Let $X, Y, Z$ denote the three complex coordinates on $\mathcal{C}^3$. The discrete group $\Delta_{27}$ is the non-abelian subgroup of SU(3) generated by the three $\mathbf{Z}^3$ transformations

$$\begin{aligned} g_1 :& \qquad (X,Y,Z) \longrightarrow (\,\omega\, X\,,\, \omega^2\, Y\,,\, Z\,) \\ g_2 :& \qquad (X,Y,Z) \longrightarrow (\,X, \omega\, Y\,, \omega^2\, Z\,) \\ g_3 :& \qquad (X,Y,Z) \longrightarrow (\,Z\,,\, X\,,\, Y\,) \end{aligned} \tag{3.18}$$

with $\omega = e^{\frac{2\pi i}{3}}$. The D3-brane theory near this singularity, derived via rules outlined below, has the same quiver data as in figure 5. It is therefore a natural conjecture that del Pezzo 8 singularities can be viewed as a deformation of this orbifold. This correspondence may be of some use, since, unlike string theory on a general del Pezzo surface, the worldsheet CFT of strings on flat space orbifolds is soluble and the D-brane boundary conditions are exactly known [7, 43]. For completeness, we briefly summarize how the above quiver data arise from the orbifold construction [9, 14].



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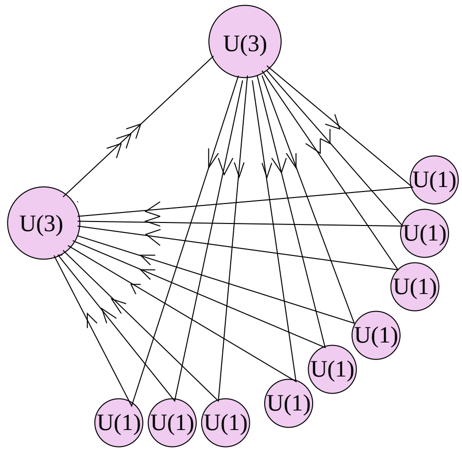


**Figure 5:** Quiver diagram of the D3-brane gauge theory on a del Pezzo 8 singularity, corresponding to the exceptional collection of fractional branes given in eq. (3.10). This is the same quiver diagram as that of the D3-brane theory near a $\mathbf{C}^3/\Delta_{27}$ orbifold singularity. The connection between the two theories is explained below.

Let $\Gamma$ be a general discrete group that acts on $\mathbf{C}^3$. Consider the D3-brane and all of its images under $\Gamma$. Their world-volume theory is a $\mathrm{U}(|\Gamma|)$ gauge theory with a vector multiplet $V$ and three chiral multiplets $\Phi_i$, that parametrize the transverse positions of the D3-branes along $\mathbf{C}^3$. The orbifold projection amounts to the requirement that

$$\begin{aligned} R_{\rm reg} V R_{\rm reg}^{-1} &= V \\ (R_3)_{ij} R_{\rm reg} \Phi^j R_{\rm reg}^{-1} &= \Phi^i \end{aligned} \tag{3.19}$$

where $R_{\rm reg}$ is the regular representation of $\Gamma$ acting on the Chan-Paton index, and $R_3$ is the 3-d defining representation. Since $R_{\rm reg}$ decomposes into irreducible representations as

$$R_{\rm reg} = \bigoplus_{a=1}^{r} n_a R^a \qquad\qquad n_a = \dim R^a. \tag{3.20}$$

the projection (3.19) breaks the gauge symmetry to $\prod_{a=1}^r \mathrm{U}(n_a)$. Translated into geometric language, we conclude that a D3-brane near an orbifold singularity splits up into fractional branes $F_a$, where $a$ labels an irreducible representation $R_a$, and that each fractional brane occurs with multiplicity $n_a = \dim R_a$. The number of chiral fields $n^3_{ab}$ transforming in the $(n_a, \overline{n}_b)$ bi-fundamental representation, is obtained by the decomposition

$$R_3 \otimes R^a = \bigoplus_{b=1}^{r} n^3_{ab} R^b. \tag{3.21}$$

The group $\Delta_{27}$ has 27 elements, that split up in 11 conjugacy classes. It also has 11 representations: nine 1-dimensional representations, and two 3-dimensional ones. The above orbifold procedure thus produces a quiver gauge theory with gauge group $\mathrm{U}(3)^2 \times \mathrm{U}(1)^9$. Using the formula (3.21), a straightforward calculation [14] shows that the bifundamental matter organizes as in the quiver of figure 5. We are thus motivated to look for a relationship between the geometry of the orbifold space $\mathbf{C}^3/\Delta_{27}$ and del Pezzo 8 surfaces. Consider the following combinations of coordinates

$$\begin{aligned} x &= XYZ \\ y &= (X^3 + \omega Y^3 + \omega^2 Z^3)(X^3 + \omega^2 Y^3 + \omega Z^3) \\ z &= X^3 + Y^3 + Z^3 \\ w &= (X^3 + \omega Y^3 + \omega^2 Z^3)^3 \end{aligned} \tag{3.22}$$

with $\omega = e^{\frac{2\pi i}{3}}$. From eqn (3.18), it is evident that these expressions are all invariant under the action of $\Delta_{27}$. Each thus defines a single-valued coordinate on the orbifold space. If we give $(X, Y, Z)$ weight $\frac{1}{3}$, then the new invariant combinations in (3.22) are homogeneous of weight (1,1,2,3). These are the same weights as of the projective space used in representing $\mathcal{B}_8$. With not too much extra work, one can indeed prove that the coordinates $(x, y, z, w)$ defined in (3.22) satisfy a homogeneous equation of the form

$$w^2 + y^3 - 27wx^3 + wz^3 - 3wyz = 0 \tag{3.23}$$

This confirms the identification of $\mathbf{C}^3/\Delta_{27}$ as a special point in the moduli space of del Pezzo 8 singularities, and (at least partially) explains the correspondence of the D3-brane gauge theories. The orbifold perspective can be useful in case one wants to verify properties of the string theory using an exact string worldsheet calculation. The general geometric description of D3-branes is limited to the large volume regime. On the other hand, as we will see shortly, it has the advantage of being a step closer to providing a purely geometrical description of the space of gauge invariant coupling constants. Ideally, of course, one would like to have both descriptions available.

### 3.4 Seiberg dual

For a given geometrical singularity, there are in principle many different exceptional collections of fractional branes. The allowed choices are typically inequivalent, and in particular lead to different world-volume gauge theories on the probe D3-brane. There exists a simple transitive set of transformations on the space of exceptional collections, known as mutations. A useful subclass of mutations has the physical interpretation of Seiberg duality [44, 33]: the $\mathcal{N} = 1$ supersymmetric gauge theories corresponding to the original and mutated set of fractional branes are each others Seiberg dual. For a given singularity, the question of which of the dual descriptions is most appropriate is determined by the value of the geometric moduli that determine the gauge theory couplings.

To apply this duality map to a given exceptional collection, one chooses a particular node $F_i$. One then orders all $F_j$ such that all branes connected to $F_i$ via incoming lines are placed to the left of $F_i$ and all others are placed to the right. Seiberg duality, applied to the node $F_i$ then amounts to the following map on the charge vectors situated to the left of $F_i$

$$\mathrm{ch}(F_j) \longrightarrow \mathrm{ch}(F_j) - \chi(F_j, F_i)\,\mathrm{ch}(F_i) \tag{3.24}$$



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As a result of this change of basis, the multiplicity of the node $F_i$ needs to be adjusted, so as to preserve the requirement that all charge vectors must add up to that of a single D3-brane. The proper adjustment is

$$n_j \;\to\; n_j - N_j \tag{3.25}$$

where the integer $N_j$ is given by the sum

$$N_j = \sum_{i<j} \chi(F_i, F_j)\, n_j. \tag{3.26}$$

We recognize $N_j$ as the number of flavors at the node $F_j$. Eqn (3.25) thus corresponds to the replacement of $N_c$ with $N_f - N_c$. This supports the interpretation of the map (3.24) as a Seiberg duality. It is also straightforward to verify that the change due to (3.24) in the number of bi-fundamentals between the nodes is completely consistent with this physical interpretation.

Geometrically, the transformation (3.24) on the basis of charge vectors can be recognized as the Picard-Lefschetz monodromy around a conifold point. There is a natural interpretation of this. The quiver theory we have discussed lives at a locus in Kähler moduli space where the del Pezzo surface has shrunk to zero size, but where string perturbation theory is still applicable. There are other places in Kähler moduli space where some cycle has shrunk to zero size and string perturbation theory breaks down — these are generalized conifold points. We can imagine traversing a loop in moduli space starting at the point where the conformal quiver theory lives, and going around a conifold point, where the central charge of a given fractional brane $F_i$ vanishes. This will implement the transformation (3.24) on the charge vectors. From the point of view of the worldvolume theory, the change in Kähler parameters translates into a change in the gauge coupling of the $\mathrm{U}(|n_i|)$ gauge group. As we go around the loop, this gauge coupling is pushed through strong coupling, and we have to do a Seiberg duality on the $i$th node.[6]

Let us specialize to the D3-brane theory on del Pezzo 8. Besides the $\Delta_{27}$ orbifold quiver theory of figure 5, we now know that it gives rise to a more general family of $\mathcal{N}=1$ gauge theories obtained via Seiberg dualities. In particular, we can apply Seiberg duality map to the fractional brane $F_{10}$ in eqn (3.10). Via (3.30), this map amounts to replacing the neighboring node $F_{11}$ by a new fractional brane $\tilde{F}_{11}$ with charge vector

$$\mathrm{ch}(\tilde{F}_{11}) \;=\; -\mathrm{ch}(F_{11}) + 3\,\mathrm{ch}(F_{10}) \;=\; \Big(3, \sum_{i=5}^{8} E_i, -2\Big) \tag{3.27}$$

As a consequence of this mutation, the multiplicity of $F_{10}$ changes from $n_{10} = 3$ to $\tilde{n}_{10} = -6$. This is as expected from the Seiberg duality map on the field theory: the original node has 9 flavors and 3 colors, and the new node therefore has $N_f - N_c = 6$ colors. The dual quiver diagram is given in figure 6. In the next section we will use this Seiberg dual quiver theory as our starting point for a open string construction of an MSSM-like gauge theory.

[6] One should take care to pick a path in moduli space such that the low energy gauge theory is still applicable and we do not have to worry about stringy corrections. It is not completely clear that this is always possible, but since the effect of monodromy can be d escribed purely in field theoretic terms, this seems quite reasonable.



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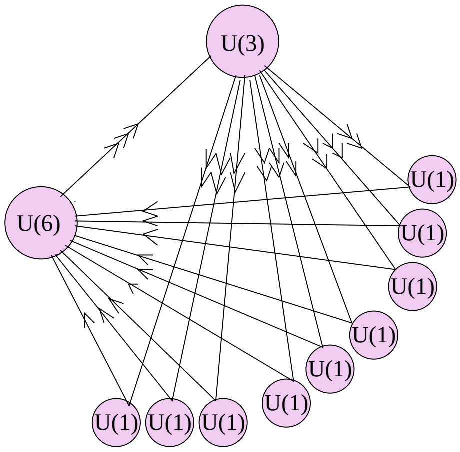


**Figure 6:** Seiberg dual of the D3-brane gauge theory on the $\Delta_{27}$ orbifold, or equivalently, of the exceptional collection of fractional branes (3.10)–(3.27) on the del Pezzo 8 singularity.

### 3.5 Superpotential

Thus far, we have focused on the topological properties of the D-brane theory. Non-topological data are harder to control and compute. There is however one more valuable piece of information that can be extracted with precision from the geometric perspective, namely the holomorphic superpotential $W$. For quiver gauge theories, $W$ is a sum of gauge invariant traces over ordered products of bi-fundamental chiral fields. In principle there is one such term for each oriented closed loop on the quiver. In the example of figure 5 or 6, it is known that $W$ is a purely cubic function:

$$W = C_{abc}\mathrm{Tr}(\phi^a \phi^b \phi^c) \tag{3.28}$$

We can compute the cubic couplings by computing disk three-point amplitudes in topological string theory. In the large volume limit, the internal part of the vertex operator for a chiral field is a generator of the Ext group between two fractional branes. The three-point functions are then proportional to the Yoneda composition of the Ext generators:

$$\mathrm{Ext}^l(i_*F_i, i_*F_j) \times \mathrm{Ext}^m(i_*F_j, i_*F_k) \times \mathrm{Ext}^{3-l-m}(i_*F_k, i_*F_i) \to \mathrm{Ext}^3(i_*F_i, i_*F_i) \equiv \mathbf{C}. \tag{3.29}$$

This calculation was done explicitly for del Pezzo singularities $\mathcal{B}_n$ with $n \leq 6$ in [35]. The superpotential resulting from this calculation is a meromorphic function of the space of complex structure deformations of the del Pezzo singularity.

Now let us specialize to the D-brane on $\mathcal{B}_8$. Let us first consider fig 5. Its superpotential $W$ is a cubic expression with $3\times 9 = 27$ terms, equal to the number of triangles of the quiver. Naively this gives 27 independent Yukawa couplings. However, since we have no direct knowledge of Kähler potential terms, we are free to perform arbitrary field redefinitions, as long as they are compatible with the structure of the quiver. The group of allowed field redefinitions is $GL(1)^9 \times GL(3)$. This reduces the number of independent parameters in $W$ to $27 - (9 + 9) + 1 = 10$. (We subtracted the overall scale of $W$.) The Seiberg dual theory of figure 6 yields to the same number parameters: there are $3 \times 18 = 54$ terms, but the group of allowed field redefinitions is $GL(2)^9 \times GL(3)$. This again gives $54 - (9 \times 4 + 9) + 1 = 10$ parameters. Eight these 10 parameters can be identified with complex structure deformations of the del Pezzo 8 surface.

### 3.6 Other geometric moduli

Besides the superpotential, the D3-brane gauge theory has many other gauge invariant couplings, which arise as closed string modes of the del Pezzo singularity. These couplings are of vital importance for making a realistic model. We will make a few comments about the correspondence below, but the full dictionary has not yet been established.

The complex structure parameters of a general Calabi-Yau surface are associated to $(2,1)$ forms and fit into $\mathcal{N}=2$ vector multiplets. For the del Pezzo singularities, the complex structure moduli that preserve the singularity correspond to $(2,1)$ forms with non-compact support.[7] Their auxiliary fields correspond to turning on various 3-form RR and NS fluxes proportional to the same $(2,1)$ forms and their complex conjugates. In the 4-dimensional Lagrangian, vector multiplet moduli appear as spurion fields in the superpotential, and turning on auxiliary fields gives rise to certain soft supersymmetry breaking terms, namely non-supersymmetric Yukawa couplings [45].

To every 2-cycle in the Calabi-Yau manifold we can associate a hypermultiplet. For the $n$-the del Pezzo singularity, there are $n+1$ 2-cycles. If we denote the two-cycle by $C_I$, then the scalars in the hypermultiplet are given by the period integrals of two-form potentials. When we put in branes, we break half of the supersymmetries and we get two $\mathcal{N} = 1$ multiplets for each 2-cycle. The closed string scalars re-arrange themselves as follows. Consider a D5 brane wrapped on the 2-cycle. Its 4-d gauge coupling is given by

$$\tau_I = \int_{C_I} \Big( C^{RR}_{(2)} - \tau B^{NS} \Big). \tag{3.30}$$

The other hypermultiplet scalars

$$\int_{C_I} (J + iC_{(4)}) \tag{3.31}$$

become linear multiplets in four dimensions. They contain the Fayet-Iliopoulos terms, and control the size of the 2-cycles. Thus hypermultiplet moduli appear as spurion fields in the D-terms. The auxiliary fields of the hypermultiplets should correspond to some interesting deformations of the background that to our knowledge have not been studied in any detail yet. Turning them on gives rise to several interesting soft SUSY-breaking terms, such as gaugino masses and other non-supersymmetric mass terms.

[7]There are other complex structure deformations of the Calabi-Yau, which are localized around the tip of the cone, that make the geometry less singular. They play a role when one considers worldvolume theories of fractional branes which confine; these complex structure parameters are then identified which gaugino condensates.

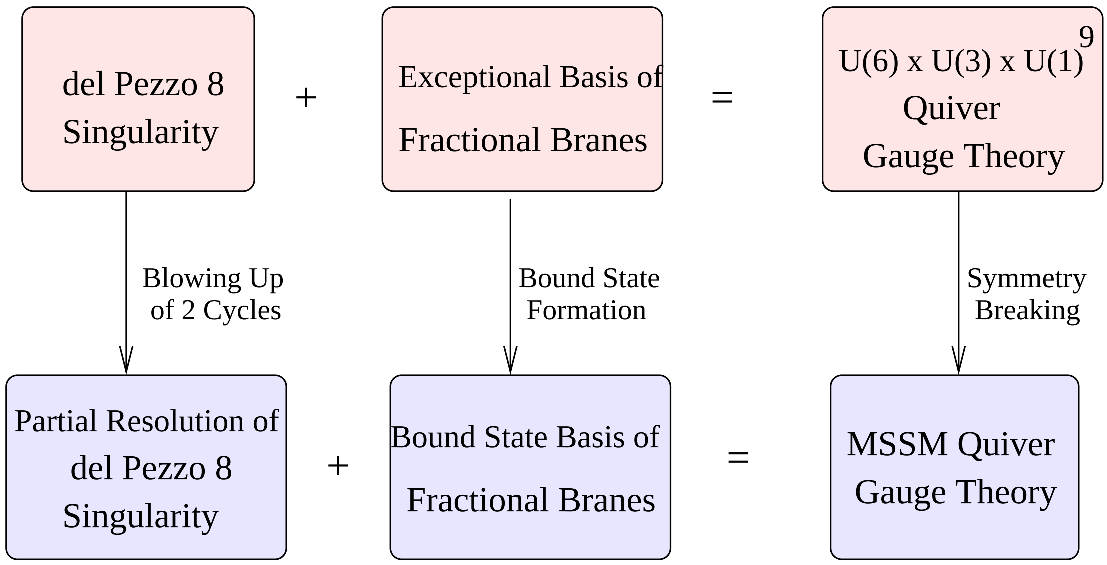


**Figure 7:** The ouline of our construction of a geometric dual of the MQSSM.

## 4. Building the standard model on a D3-brane

We will now look for suitable symmetry breaking process, such that the left-over low energy theory, ideally, looks like the supersymmetric Standard Model, or as a slightly more modest target, like its minimal quiver extension: the MQSSM quiver gauge theory introduced in section 2. The concrete plan is as follows (see figure 7): starting from the D3-brane theory on $\mathcal{B}_8$, we choose a suitable configuration of expectation values of bi-fundamental scalar fields. We then translate the symmetry breaking process in the geometric language of bound state formation of fractional branes. In this way we manufacture a new basis of fractional branes, characterized by their collection of charge vectors, such that the corresponding D3-brane gauge theory reproduces the MQSSM theory. The new basis of fractional branes will live on a partially resolved del Pezzo 8 singularity.

The minimal quiver theory of a D3 on $\mathcal{B}_8$, as given in figure 5, was the starting point for a string construction of the MSSM proposed in [21]. It was argued in [21] that, by turning on FI-terms, one can induce the condensation of bi-fundamentals that connect 3 of the $\mathrm{U}(1)$-factors with one of the $\mathrm{U}(3)$ nodes, thereby breaking $\mathrm{U}(3)$ to $\mathrm{U}(2)$. Inspection of the full set of D-term equations, however, shows that one can not break one of the $\mathrm{U}(3)$'s without also breaking the other. This is not what we want, since we need an unbroken SU(3) color group. For this reason, we will start from the Seiberg dual theory with gauge group $\mathrm{U}(6) \times \mathrm{U}(3) \times \mathrm{U}(1)^9$. Its quiver diagram is drawn in figure 6.

We will now show that the quiver gauge theory of figure 6 can indeed be reduced to the MQSSM quiver theory of figure 2. Our main assumption will be that we have complete freedom to tune all gauge invariant coupling constants: the FI-terms, the Yukawa couplings, as well as the gauge couplings.

### 4.1 Symmetry breaking to $\mathrm{U}(3)\times\mathrm{U}(2)\times\mathrm{U}(1)^7$

We denote the three types of bifundamental fields, as

$$X^p \in (1,\bar{6})\,, \qquad Z^q \in (6,\bar{3})\,, \qquad U^{p,r} \in (3,\bar{1}) \tag{4.1}$$

The label $p$ runs from 1 to 9, and $q$ runs from 1 to 3, while $r$ runs from 1 to 2. The general (leading order) form of the superpotential is

$$W = \sum_{p,q,r} C_{pqr}\, X^p Z^q U^{p,r} \tag{4.2}$$

and the abelian D-term equations are

$$\begin{aligned} \sum_r |U^{p,r}|^2 \;-\; |X^p|^2 &= \zeta_p \qquad p=1,..,9 \\ \sum_p |X^p|^2 - \sum_q |Z^q|^2 &= \zeta_{10} \\ \sum_q |Z^q|^2 - \sum_{p,r} |U^{p,r}|^2 &= \zeta_{11} \end{aligned} \tag{4.3}$$

Now by assumption, we allow general deformations of the closed string background, and we are thus free to arbitrarily tune the couplings in the superpotential, as well as the FI-terms. For the moment we will assume that the superpotential vanishes, so that we can ignore the F-flatness conditions. The expectation values of the scalar fields are then determined via the above equations (4.3) in combination with the non-abelian D-term equations (here $T^a$ and $t^b$ indicate the SU(6) and SU(3) generators)

$$\begin{aligned} \sum_p \bar{X}^p\, T^a X^p &= \sum_q \bar{Z}^q\, T^a Z^q \\ \sum_q \bar{Z}^q\, t^b\, Z^q &= \sum_{p,r} \bar{U}^{p,r}\, t^b U^{p,r} \end{aligned} \tag{4.4}$$

After turning on the FI-parameters, the abelian D-term equations dictate that at least some of the bi-fundamental fields condense. The non-abelian D-flatness equations (4.4) make clear that the condensation must simultaneously occur in all three groups of bi-fundamentals. We choose the following special form of the expectation values (here we only write the fields with non-zero VEVs):

$$X^1 = \begin{pmatrix} \phi_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \qquad X^2 = \begin{pmatrix} 0 \\ \phi_2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \qquad X^3 = \begin{pmatrix} 0 \\ 0 \\ \phi_3 \\ 0 \\ 0 \\ 0 \end{pmatrix},$$

$$U^{1,r} \;=\; (\,\chi^r, 0, 0\,) \tag{4.5}$$

$$Z^1 = \begin{pmatrix} \psi_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad Z^2 = \begin{pmatrix} 0 & 0 & 0 \\ \psi_2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad Z^3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \psi_3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



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This choice breaks the gauge symmetry to

$$\mathrm{U}(3) \times \mathrm{U}(2) \times \mathrm{U}(1)^7 \tag{4.6}$$

One can always adjust the FI-parameters $\zeta_i$ such that this choice of expectation values solves the abelian D-term equations. It is not difficult to show that the non-abelian D-term equations (4.4) are also satisfied, for suitable choice of $\phi_n$, $\psi_n$ and $\chi_r$ (see appendix).

After turning on the superpotential, the F-flatness equations will impose additional restrictions. For a general choice of $W$, these may not be solved by the above configuration of vacuum expectation values. In this case, the theory may need to choose another symmetry breaking pattern or supersymmetry may be broken. We will assume, however, that we have sufficient control over all parameters to ensure that the above expectation values are compatible with F-flatness. Specifically, we assume that all Yukawa couplings that connect two fields with a non-zero VEV can be tuned to zero. This amounts to the conditions (for notation see eqn (4.2))

$$C_{1,q,r} = C_{q,q,r} = 0 \qquad\qquad {\scriptstyle q = 1,2,3;\;\; r = 1,2.} \tag{4.7}$$

The remaining non-zero Yukawa couplings then typically result in mass-terms for the matter fields, proportional to their coupling to the vacuum condensates. We would like to determine the typical matter content that survives in the low-energy theory. We could of course proceed to study this question from the gauge theory point of view. Instead, however, let us first return to the geometric description in terms of fractional branes, since this provides a useful dual perspective.

### 4.2 Geometric derivation of the low energy theory

The unbroken gauge theory corresponds to the exceptional collection of fractional branes with charge vectors as given in (3.10), with $F_{11}$ replaced by $\tilde{F}_{11}$ in (3.27). In order to trigger the symmetry breaking we switched on certain FI-terms; by turning on large VEVs and integrating out very massive modes, we reduce to a simpler quiver theory. From the geometric perspective, turning on FI-terms corresponds to turning on certain blow-up modes which partially resolve the singularity. The Higgsed down quiver theory is the worldvolume theory for a D3-brane probing this simpler, partially resolved singularity.

A basis of fractional branes for the simpler singularity may be obtained from the fractional branes of the original singularity. The intuitive picture is that turning on VEVs in the quiver theory corresponds to bound state formation of fractional branes. Of course, we have been using this idea all along, because our quiver theory is just a way of describing our probe D3 brane as a bound state of fractional branes.

To describe this condensation process in terms of sheaves on collapsing cycles[8] can be somewhat complicated. However it can be very simply described at the level of charge vectors. When two fractional branes $F_1$ and $F_2$ bind into $F_B$, the corresponding nodes in

[8]Or complexes of such sheaves.

the quiver diagram collapse to one. The charge vector of the bound state associated to the new node is the sum of the two constituents

$$\mathrm{ch}(F_B) = \mathrm{ch}(F_1) + \mathrm{ch}(F_2)\,, \tag{4.8}$$

So it is relatively straightforward to obtain the charge vectors associated to each node in the new quiver diagram, and hence this is a simple method to determine the net field content after condensation.

The pattern of bound state formation in our model follows by inspection of the set of expectation values (4.5). The rule we will follow is that all fractional branes (= gauge group factors) that are connected by matter fields with a non-zero expectation value are part of the same bound state. Applying this rule, we arrive at the following charge vector of the bound state:

$$\mathrm{ch}(F_0) = 3\,\mathrm{ch}(F_{10}) - \sum_{i=1,2,3} \mathrm{ch}(F_i) - \mathrm{ch}(\tilde{F}_{11})$$

which gives

$$\mathrm{ch}(F_0) = \Big(3, -2K + \sum_{i=5}^{8} E_i - E_4, \tfrac{1}{2}\Big) \tag{4.9}$$

As a result, the new basis of fractional branes is $(F_0, F_4, \ldots, F_9, F_{10}, \tilde{F}_{11})$. The respective multiplicities are $(1, 1, \ldots, 1, -3, 2)$, in accordance with (4.6).

The number of oriented lines that connect to the new node are obtained by adding or subtracting, depending on the relative orientation, all the lines that connect to the original two nodes. These reduction rules for eliminating nodes and lines from the quiver diagram properly reflect the lifting of gauge fields and bi-fundamental matter from the low energy theory. To determine the matter content, we compute the intersection pairings according to the formula (3.14). The respective ranks of the fractional branes are $(3, 1, \ldots, 1, 3, 3)$ and the respective degrees are $(5, 2, \ldots, 2, 5, 4)$. We thus obtain the following intersection matrix

$$\chi_-(F_i, F_j) = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & -3 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -2 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -2 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -2 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -2 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -2 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -2 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & -3 \\ 3 & 2 & 2 & 2 & 2 & 2 & 2 & 3 & 0 \end{pmatrix} \tag{4.10}$$

The resulting quiver diagram is depicted in fig 8. We organized the bifundamental matter content into three generations of quarks and leptons. The quiver looks very similar to the minimal quiver extension of the MSSM as drawn in figure 2. The only difference is that two of the $\mathrm{U}(1)$ nodes are replaced by $\mathrm{U}(1)^3$. We will discuss possible ways of eliminating these extra $\mathrm{U}(1)$'s in the subsection 4.4.

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In eqn (4.10), $\chi_-(F_i, F_j)$ denotes the anti-symmetric part of the the Euler character: it counts the number of bifundamentals from $F_i$ to $F_j$ minus those from $F_j$ to $F_i$. However, the new basis of fractional branes is no longer an exceptional collection, and it would therefore be possible that lines of both orientations appear between two nodes. To obtain more information, let us compute the full formula (3.13) for the Euler character. [9] We obtain

$$\chi(F_i, F_j) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 2 & 2 & 2 & 2 & 2 & 2 & 3 & 1 \end{pmatrix} \tag{4.11}$$

Let us make some comments on this result. First, the fact that all diagonal elements are equal to 1 is good news: the 1 represents the gauge multiplet of the corresponding node, and thus indicates the absence of any adjoint matter. We further see that, although this set of fractional branes cannot correspond to an exceptional collection, the abundance of 0's in off-diagonal entries shows that it comes very close. This is desirable, since it gives a strong indication that there are no extra fields beyond the ones exhibited in the quiver diagram of figure 8.

### 4.3 A field theory check

As an independent check on the geometric calculation, we can try to obtain the spectrum and interactions after symmetry breaking from the field theory perspective. Based on the form (4.5) of the condensates, as well as the structure of the quiver in figure 8, we propose that the MSSM fields are obtained from the original quiver fields as follows

$$X^i = \begin{pmatrix} * \\ * \\ * \\ * \\ * \\ * \end{pmatrix}, \qquad X^{i+3} = \begin{pmatrix} \bar{\nu}_i \\ * \\ * \\ \bar{u}_i \\ \bar{u}_i \\ \bar{u}_i \end{pmatrix}, \qquad X^{i+6} = \begin{pmatrix} \bar{e} \\ * \\ * \\ \bar{d}_i \\ \bar{d}_i \\ \bar{d}_i \end{pmatrix},$$

$$U^{i,r} = (\,*,\ *,\ *\,) \qquad U^{i+3,r} = (\,*\,, H_u^{i,r}, H_u^{i,r}\,) \qquad U^{i+6,r} = (\,*\,, H_d^{i,r}, H_d^{i,r}\,)$$

$$Z^1 = \begin{pmatrix} * & L_1 & L_1 \\ * & * & * \\ * & * & * \\ * & Q_1 & Q_1 \\ * & Q_1 & Q_1 \\ * & Q_1 & Q_1 \end{pmatrix} \qquad Z^2 = \begin{pmatrix} * & * & * \\ * & L_2 & L_2 \\ * & * & * \\ * & Q_2 & Q_2 \\ * & Q_2 & Q_2 \\ * & Q_2 & Q_2 \end{pmatrix} \qquad Z^3 = \begin{pmatrix} * & * & * \\ * & * & * \\ * & L_3 & L_3 \\ * & Q_3 & Q_3 \\ * & Q_3 & Q_3 \\ * & Q_3 & Q_3 \end{pmatrix}$$

[9]Evaluation of the integral (3.13) gives:

$$\chi(F_i, F_j) = \mathrm{rk}(F_i)\mathrm{rk}(F_j) + \mathrm{rk}(F_i)\mathrm{ch}_2(F_j) + \mathrm{ch}_2(F_i)\mathrm{rk}(F_j) - c_1(F_i) \cdot c_1(F_J) + \frac{1}{2}\chi_-(F_i, F_j)$$



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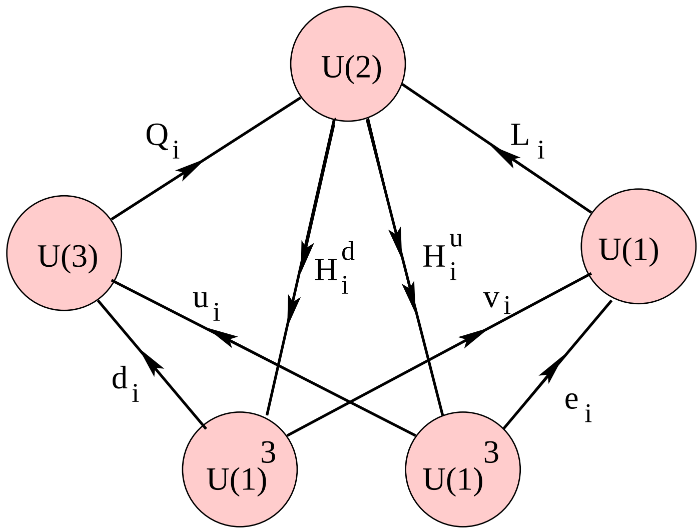


**Figure 8:** The quiver diagram of the collection of fractional branes given in eqns (3.10) and (4.9). It has the same matter content and gauge symmetry as the MQSSM given in figure 2, except that each of the three generations couples to a different $\mathrm{U}(1)$ factor.

with $i = 1, 2, 3$. All entries not indicated with a $*$ have, due to the constraints (4.7) on the superpotential, no direct Yukawa coupling to the fields with non-zero expectation value. These components are thus expected to remain massless after the symmetry breaking. Conversely, all fields marked by the $*$'s are expected to acquire a mass, either via Yukawa coupling to a field with non-zero VEV or by being eaten via the Higgs mechanism.[10]

While it would be useful to analyze the symmetry breaking process is more detail, we like to emphasize that we prefer to view the original high energy quiver theory as only an intermediate step towards a direct string construction of the unbroken MSSM-like theory.

### 4.4 Decoupling of $\mathrm{U}(1)$ symmetries

The quiver diagram of figure 8 displays a total of nine $\mathrm{U}(1)$ factors. Most of these, however, are automatically or easily decoupled from the low energy theory. First, since all fields are neutral under the overall $\mathrm{U}(1)$ symmetry $Q_{\mathrm{tot}} = \sum_i Y_i$, this overall factor decouples. Secondly, it can easily be shown that there are two $\mathrm{U}(1)$ factors that have mixed anomalies with the non-abelian gauge symmetries. In the full string theory realization of the quiver gauge theory, these anomalies are cancelled via a generalized Green-Schwarz mechanism, which in addition renders the corresponding $\mathrm{U}(1)$ vector bosons massive. In the decou-

[10] A straightforward accounting exercise shows that, in fact, the leading order cubic superpotential (4.2) is not sufficient to lift all fields with a $*$. To make all fields massive, one need to include higher order terms in $W$. These "irrelevant" terms can become relevant after turning on the expectation values (4.5)–(4.5).

pled low energy theory, these U(1)'s thus survive as anomalous global symmetries. For completeness, let us briefly outline the relevant stringy cancellation mechanism [46].

The world-volume action of fractional Dp-branes includes a CS-coupling to the RR-potentials of the form

$$\int C^{(p-1)} \wedge \mathrm{Tr} F \ , \qquad \text{and} \qquad \int C^{(p-3)} \wedge \mathrm{Tr}(F \wedge F) \ . \tag{4.12}$$

Upon integrating out the $C$-fields, these interaction terms give rise to additional anomalous contributions to the D3-brane gauge theory action that cancel the quantum mechanical anomalies, due to the presence of chiral fermions. The compensating contributions arise from the coupling, via the C-field propagator, between the first and second type of interaction terms in eqn (4.12). The coupling between two interaction terms of the first type in (4.12) gives rise to the Stueckelberg type mass-terms for the vector bosons. In the decoupled theory, only the gauge bosons of the anomalous U(1) factors acquire a mass. An intuitive explanation[11] for this is that the anomalous U(1) factors are in one-to-one correspondence with fractional branes that wrap cycles that, via the intersection pairing, are dual to *compact* cycles within the non-compact Calabi-Yau geometry. There are two such cycles: the 4-cycle that wraps the del Pezzo 8 surface, and its dual degree one 2-cycle. By contrast, all degree zero 2-cycles within the del Pezzo surface are dual to non-compact 4-cycles. It is therefore natural that the associated closed string modes, which are the would-be longitudinal components of the non-anomalous U(1)'s, have non-normalizable kinetic terms. The vector bosons of the anomaly free U(1)'s thus survive as massless low energy degrees of freedom in the non-compact theory.

This leaves us, of the original nine, with a total of six U(1) gauge symmetries. Their generators can be recognized as the hypercharge $Y$ and $B-L$, as identified for the MQSSM in section 2, and four additional charges, given by the difference of two generators within each $\mathrm{U}(1)^3$ node in figure 8. There are several ways in which the extra U(1)'s can be decoupled from the low energy physics. For instance, it is natural to assume that the superpartners of the right-handed neutrinos acquire a non-zero expectation value. This VEV breaks three of the U(1) symmetries. Turning on generic Higgs expectation values will then break all the remaining U(1)'s, *except* for the electro-magnetic gauge group generated by $Q_{EM} = Y + T_3$. In other words: simply by assuming non-trivial sneutrino and Higgs VEVs, our model automatically leads to the correct electro-magnetic charge assignments of quark and leptons.

For the purpose of reproducing the MQSSM of figure 2 from our quiver in figure 8, one could in principle attempt a direct approach and decouple the extra U(1) vector bosons from all observable matter, simply by tuning the corresponding U(1) coupling constants and make them sufficiently small. The extra U(1)'s then become global symmetries, that forbid certain undesirable generation mixing couplings. We plan to study the fate of the U(1) symmetries after embedding of our construction inside a compact Calabi-Yau manifold in an upcoming paper.

[11]It would be worthwhile to work out the following argument in more detail.



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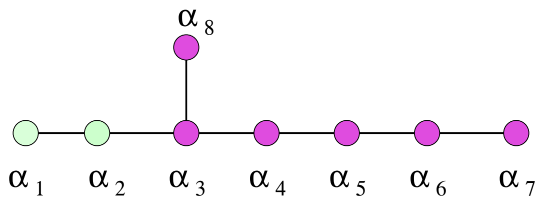


**Figure 9:** The symmetry breaking towards the MQSSM of figure 8 is geometrically dual to blowing up the 2-cycles $\alpha_1$ and $\alpha_2$, corresponding to the first two roots of $E_8$, of the del Pezzo 8 surface. In order to perform the blow-up, the complex structure is tuned so that the del Pezzo develops an $A_2$ singularity.

### 4.5 Geometric dual of the MQSSM

With some hindsight, we can now give a more intrinsic geometric characterization of the reduced geometry and collection of fractional branes described by the MQSSM, from which any reference to the symmetry breaking process and bound state formation has been erased. Notice that the new bound state basis of fractional branes, as given in eqns (3.10), (3.27) and (4.9), does not contain the 2-cycles $\alpha_1$ and $\alpha_2$ (see eqn (3.4)). This indicates that these 2-cycles have been removed from the singularity by blowing them up.

Being cycles with self-intersection $-2$, the $\alpha_i$ in general do not appear as effective curves: they can be blown up only at special points in the complex structure moduli space, at which the del Pezzo 8 surface develops a suitable A-D-E type singularity. In our case, we first need to tune the complex structure to obtain an $A_2$ singularity. This tuning is presumably the geometric equivalent of the requirement (4.7) on the superpotential $W$. At this special locus in complex structure moduli space, the Calabi-Yau singularity becomes degenerate and the quiver moduli space develops new branches. Then, the pair of 2-cycles $\alpha_1$ and $\alpha_2$ may be be blown up and removed from the singularity. The remaining CY singularity is our proposed geometric dual to the MQSSM.[12]

## 5. Discussion

We have identified a Calabi-Yau singularity on which the D3-brane world-volume theory reproduces the MQSSM theory of figure 2. In terms of the quarks, leptons, and gauge bosons, it has the exact same matter content as the MSSM. There are however a number of additional Higgs fields, as well as a possibly a number of (arbitrarily weakly coupled) extra U(1) gauge factors. In general, the appearence of extra Higgses and U(1)'s is characteristic of many string theoretic models, as well as to many other proposals for extending the

[12]Since the Standard Model is not a strongly coupled large $N$ gauge theory, the dual classical geometric description has somewhat limited validity, since string corrections are bound to be important. As a purely theoretical exercise, one could imagine taking a large $N$ limit of the MQSSM, by considering a large number of D3-branes on the partially resolved del Pezzo 8 singularity. After accounting for the backreaction and taking a decoupling limit, this leads to an AdS/CFT type dual geometry, which for large 't Hooft coupling is arbitrarily weakly curved. We expect this dual geometry to be directly related to our del Pezzo 8 surface with an $A_2$ singularity, presumably via a geometric transition analogous to [47].

Standard Model. In all cases, it is important to have the freedom to tune masses and couplings, to ensure compatibility with current observational bounds. Whether the present model can deal with these challenges needs further study, but the ability to (fine-)tune couplings is one of the main strengths of our set-up.

A crucial part of supersymmetric model building is the understanding of how supersymmetry gets broken. In general, supersymmetry breaking is parameterized by means of soft terms, and each mechanism generates its own characteristic pattern. In our set-up, a certain class of soft SUSY breaking terms can be geometrically understood as the effect of turning on IASD three form flux [48, 45]. As mentioned earlier in section 3.6, these fluxes appear as auxiliary fields of complex structure moduli in the superpotential and thus turning them on gives rise to non-supersymmetric Yukawa couplings. Non-SUSY mass terms can be generated by auxilary fields of closed string hypermultiplets. Unfortunately, their geometric meaning is not well understood at present, and since they appear in D-terms, their couplings to world-volume fields are much harder to compute. Nonetheless, it is evidently worthwhile to develop a better control and understanding of supersymmetry breaking in our set-up. In this respect, an interesting recent development is the realization that adding an extra vanishing del Pezzo with fractional branes to our set-up introduces a hidden sector in which supersymmetry may be dynamically broken [49].

Finally, let us address some possible criticisms of our bottom-up approach to string phenomenology. Right from the start, one could ask why it would even be useful to try to construct the Standard Model via a decoupling limit on one or more D3-branes. Since the decoupled theory has continuous parameters, the approach does not seem to be restrictive enough to lead to a phenomenologically predictive framework – at least not much beyond that of ordinary quantum field theory. Our point of view, however, is that the open string approach is a very reasonable first step towards the larger goal of string phenomenology. Eventually, QFT breaks down at the Planck scale, and string theory is our best chance of finding a fully consistent UV completion. To find out which closed string theory is the right one, it is useful to know how our present knowledge of the Standard Model translates into the geometric language of string theory. This is what we have tried to investigate. An inevitable hurdle in this quest is that the geometric language has somewhat limited validity in the regime of interest. As we hope to have shown, however, useful lessons can still be learned by trying to match the two perspectives.

Our construction of the MSSM-quiver gauge theory of figure 8 depends on several seemingly arbitrary choices. One obvious question is whether one can obtain similarly quasi-realistic gauge theories by starting with one of the lower del Pezzo surfaces or with a different class of CY singularity. The del Pezzo examples are a particularly simple class of singularities, because they have only one single collapsing 4-cycle. For our present study, we settled on the $dP_8$ example because it is sufficiently rich and because of its direct relation with the $\Delta_{27}$ orbifold. However, one can easily imagine arriving at an equivalent gauge theory via an alternate route, starting from a different geometry and via a different symmetry breaking process. On the other hand, we expect that if the final D3-brane gauge theory is the same, the final geometric singularity must also be the same. We expect that, in this respect, our bottom-up approach is robust.



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What does our model add to the many other D-brane constructions of Standard Model like field theories [16, 18 − 20]? A key distinction between our model and almost all other existing proposals, is that in our case all D-branes are localized near a very small neighborhood of the compactification manifold. Our intuitive picture is that the tip of the singularity sits a the bottom of a warped region, created via back reaction of the brane or by 3-form flux. The warp factor is such that the energy scale increases with the radial distance from the tip, which thus becomes identified with an RG parameter. The low energy open string dynamics is confined to a small, typically highly curved region near the tip and can therefore be controllably disentangled from the dynamics of the closed string moduli, that determine the global shape of the string compactification. In this way we can cleanly separate the question of closed string moduli stabilization from that of building a realistic low energy field theory. We view that as an important advantage of the bottom-up perspective.

An important possible objection to our set-up is that it seems to ignore the lesson of the unification of coupling constants. Although, by choosing to work with a single D3-brane, our specific construction does achieve some form of geometric unification, we see no obvious reason why, in our model, the couplings would need to converge at some high energy scale. Gauge unification is indeed somewhat at odds with the bottom-up philosophy, but we prefer to see the two viewpoints as complementary rather than incompatible.[13]

A final comment: it is clear that our bottom-up perspective is initially much more modest than the standard top down approach to string phenomenology. Reproducing the Standard Model in terms of a decoupled open string theory is an important near-term goal, but only a first step in the full program of string theory. It is in general a very non-trivial challenge to lift a local D-brane theory near a singularity to a fully consistent string compactification. We plan to address some aspects of this problem in the near future.


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## A. Verification of D-term equations

As promised, let us verify that the expectation values (4.5)–(4.6) satisfy all necessary D-term equations. We assume that the abelian D-term equations are satisfied by suitable

[13]A related point is that, by directly aiming at the SM rather than some grand unified gauge theory, we in principle have the freedom to choose the string scale as low as a couple of TeV or so. This has not been our main philosophy, however.



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adjustment of the FI-parameters. The SU(6) D-flatness condition requires that

$$\sum_p (X^p)^\dagger \otimes X^p = \begin{pmatrix} |\phi_1|^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & |\phi_2|^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & |\phi_3|^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{A.1}$$

equals

$$\sum_{q,i} (Z_i^q)^\dagger \otimes Z_i^q = \begin{pmatrix} |\psi_1|^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & |\psi_2|^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & |\psi_3|^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{A.2}$$

where $i$ denotes the U(3) index of $Z^q$. So we find that this equality is satisfied provided we choose $\phi_n$ equal to $\psi_n$. Similarly, the SU(3) equations require that

$$\sum_{q,I} (Z_I^q)^\dagger \otimes Z_I^q = \begin{pmatrix} \sum_{q=1}^{3} |\psi_q|^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{A.3}$$

where $I$ denotes the U(6) index of $Z^q$, equals

$$\sum_{p,r} (U^{p,r})^\dagger \otimes U^{p,r} = \begin{pmatrix} \sum_{r=1}^{2} |\chi_r|^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{A.4}$$

This equality fixes the value of $\sum |\chi_r|^2$. So we conclude that (4.5)–(4.6) indeed represents a valid vacuum, provided the superpotential $W$ is adjusted appropriately.

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